Computer-Aided Extraction of Small-Signal Model Parameters of Heterojunction Bipolar Transistors

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- An approach to parameter extraction of small-signal model of heterojunction bipolar transistors (HBT’s) is proposed in the present paper based on *S*-parameter measurements.
- Exact closed-form equations are used for the direct extraction of circuit elements. The method is characterized by its simplicity and ease of implementation.
- The *Cadence PSpice* simulator and the graphical analyzer *Cadence Probe* are used to automate the extraction procedure.
- The parameter extraction of intrinsic elements of the model is realized using macro-definitions in *Probe*. The corresponding macros in the graphical analyzer Probe are presented realizing the proposed extraction procedure.

Description of the Parameter Extraction Procedure

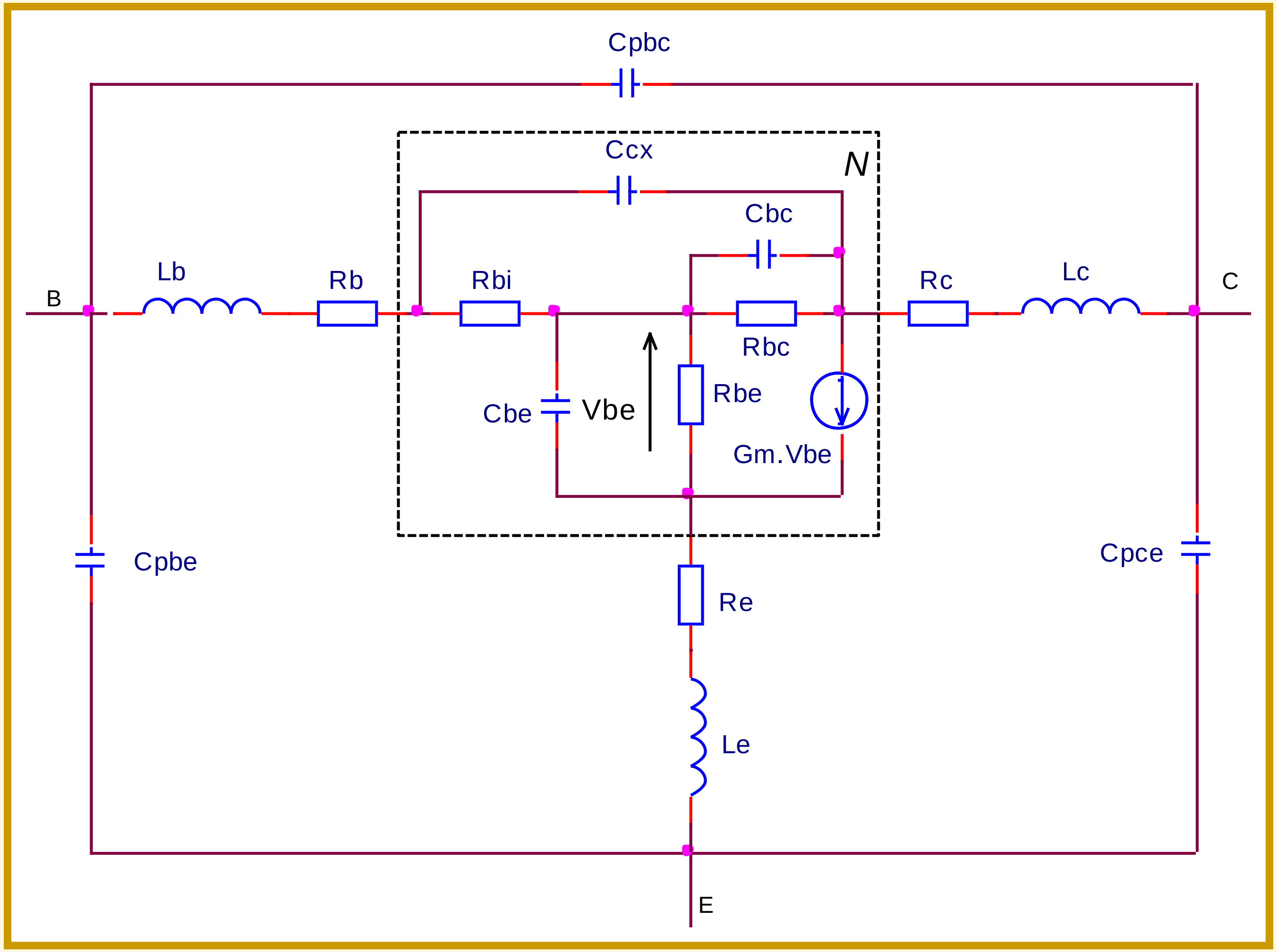
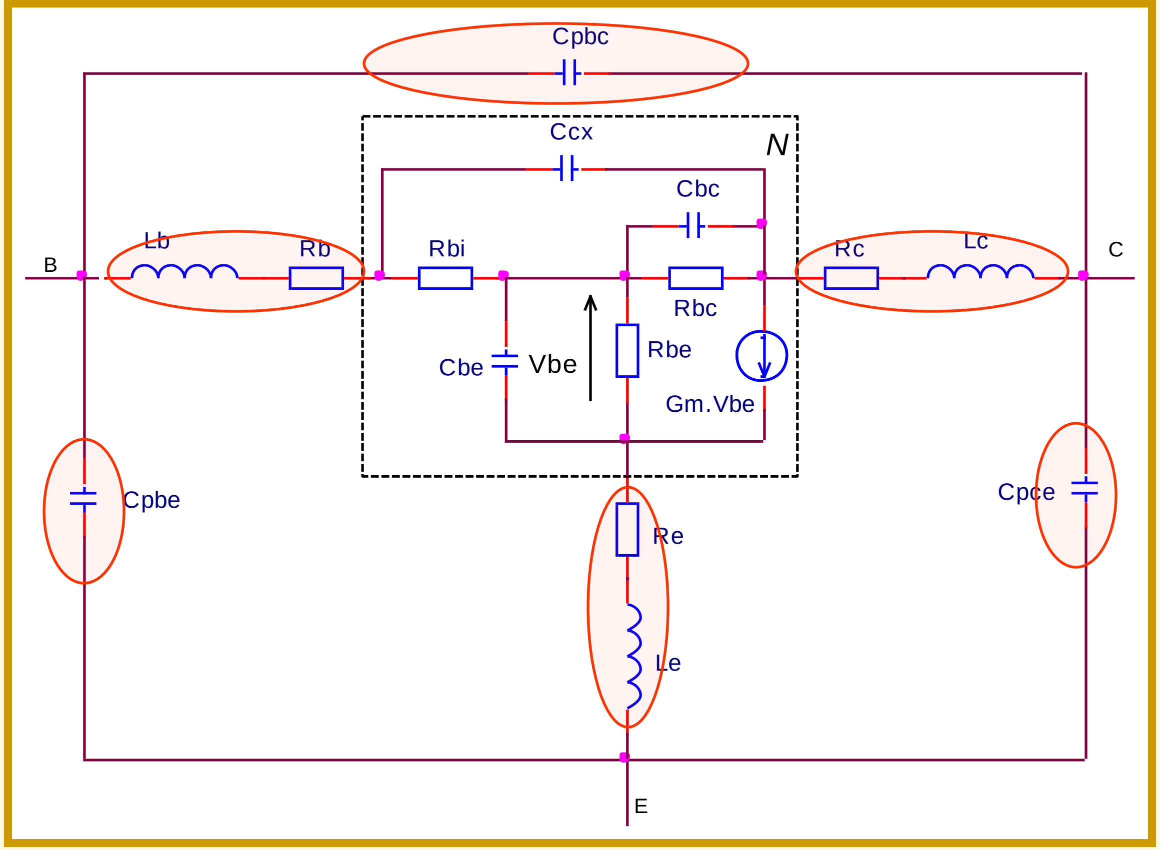


Fig. 1. Small-signal model of a HBT

Description of the Parameter Extraction Procedure

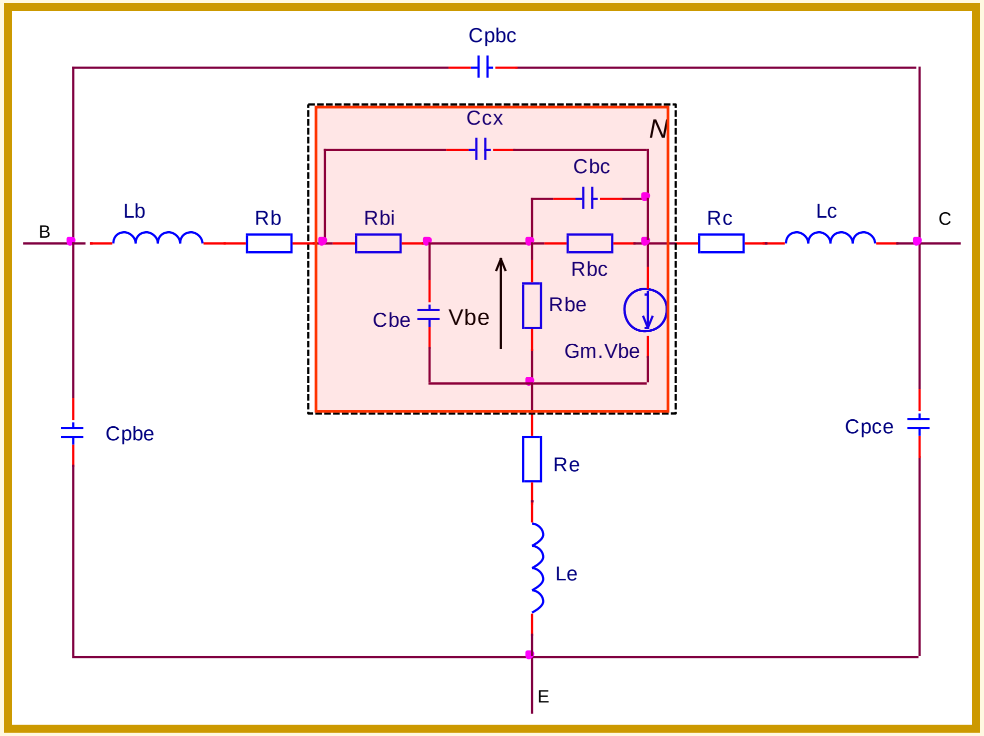
The direct extraction procedure consists of the following steps:

- De-embedding parasitic capacitances, inductances and resistances from the measured two-port *S*-parameters;



Description of the Parameter Extraction Procedure

- Conversion of the measured *S*- to *Y*- two-port parameters;
- Determination of Rbe and Cbe;
- Determination of Rbi;
- Determination of Rbc and Cbc;
- Determination of Ccx, Gmo and τ .



A. Introducing the S-parameters

- The *S*-parameters are defined using voltage controlled current sources VCCS of GFREQ type (Fig. 2). The *S*-parameters are defined using voltage controlled current sources VCCS of GFREQ type.

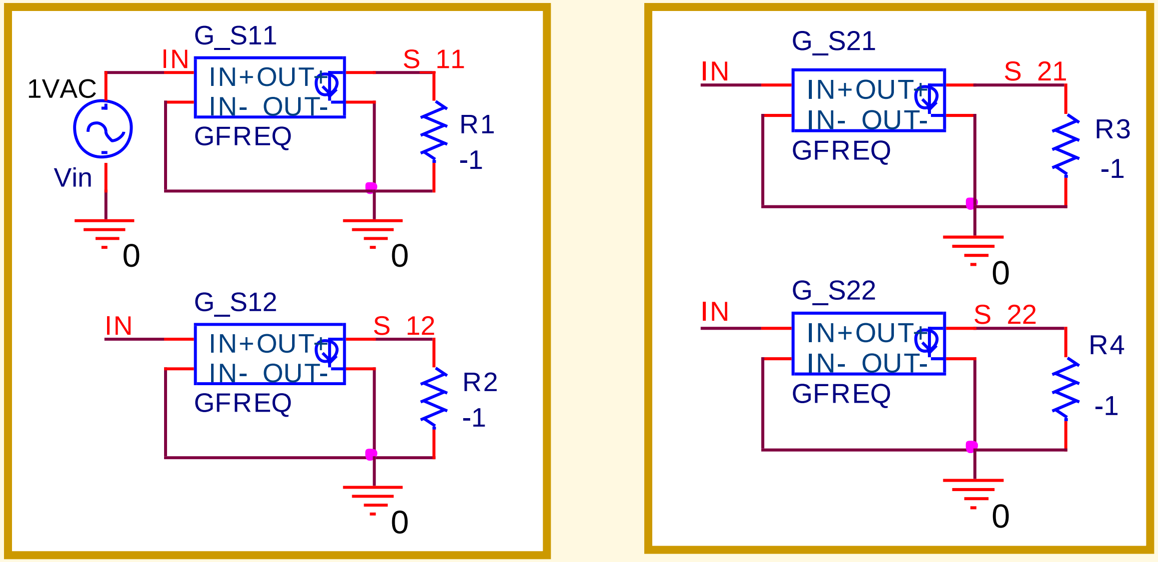


Fig. 2

A. Introducing the S-parameters

- The table has the form:

  <frequency,magnitude [DB], phase [DEG]>:

  (freq1, mag1, phase1), (freq2, mag2, phase2) …

- AC analysis is performed in the defined frequency range. The S-parameters are obtained using the following macros in the graphical analyzer Probe:

  S11 = V(S_11)

  S21 = V(S_21)

  S12 = V(S_12)

  S22 = V(S_22)

B. Conversion of S- to Y- two-port parameters

$$\boxed{Y_{11} = \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{\Delta}} \tag{1}$$

$$\boxed{Y_{22} = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{\Delta}} \tag{2}$$

$$\boxed{Y_{12} = \frac{-2S_{12}}{\Delta}} \qquad \boxed{Y_{21} = \frac{-2S_{21}}{\Delta}} \tag{3}$$

$$\boxed{\Delta = ((1+S_{11})(1+S_{22})-S_{12}S_{21})R_o}$$

B. Conversion of S- to Y- two-port parameters

The conversion is performed using the following macros in the graphical analyzer *Probe*:

```
Ro=50
delta =((1+S11)*(1+S22)-S12*S21)*Ro
Y11 = ((1-S11)*(1+S22)+S12*S21)/delta
Y12 = -2*S12/delta
Y21 = -2*S21/delta
Y22 = ((1+S11)*(1-S22)+S12*S21)/delta
```

The intrinsic part N of the small-signal transistor model of a HBT is shown in Fig. 3. Applying T-Π transformation for the subcircuit N1, the equivalent circuit shown in Fig. 4 is obtained.

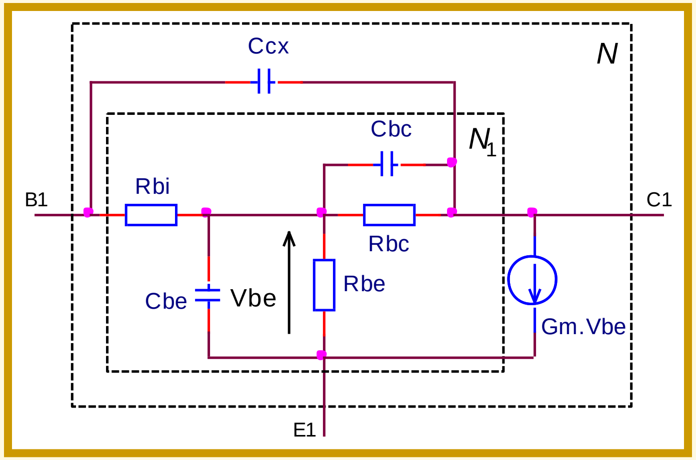


Fig. 3

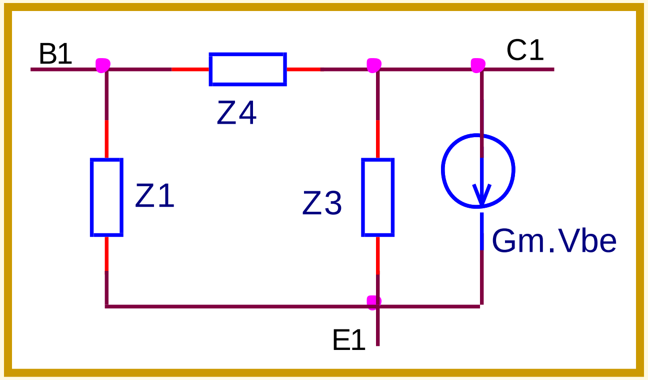


Fig. 4

$$Z_1 = \frac{D}{Z_{bc}} \; ; \; Z_2 = \frac{D}{Z_{be}} \; ; \; Z_3 = \frac{D}{Z_{bi}} , \tag{4}$$

$$Z_{bi} = R_{bi} \; ; \; Z_{bc} = \frac{R_{bc}}{1 + j\omega R_{bc} C_{bc}} \; ; \; Z_{be} = \frac{R_{be}}{1 + j\omega R_{be} C_{be}} , \tag{5}$$

$$D = Z_{bi} Z_{bc} + Z_{be} Z_{bc} + Z_{bi} Z_{be} \tag{6}$$

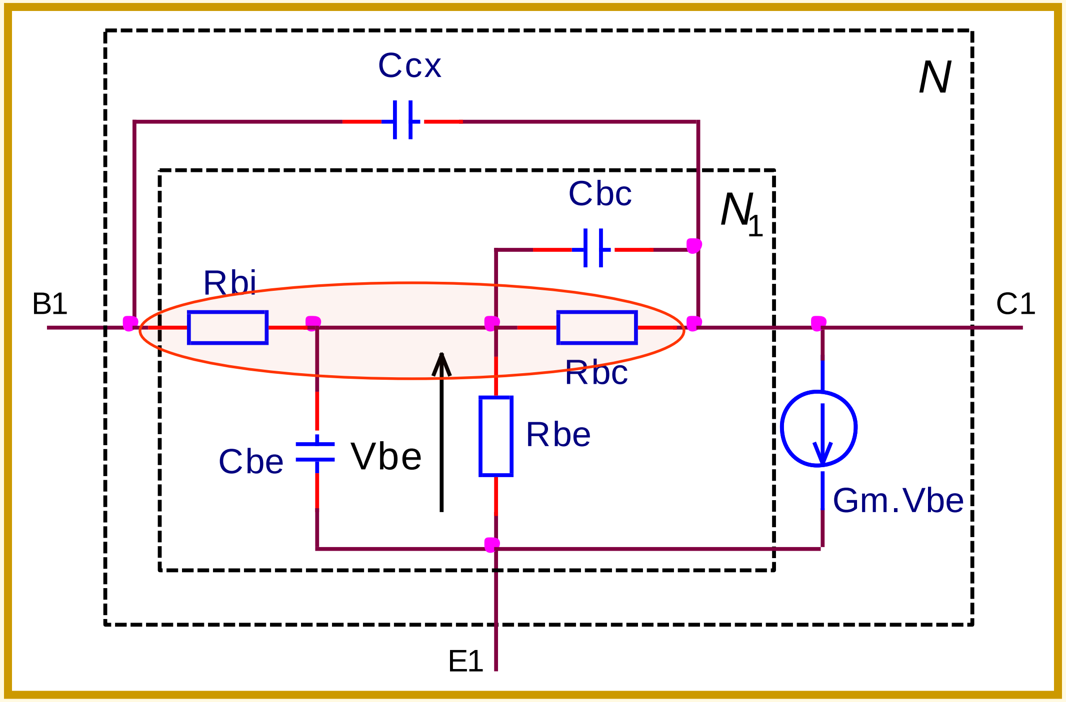
$$Z_4 = \frac{Z_2 Z_{cx}}{Z_2 + Z_{cx}} \; ; \; Z_{cx} = \frac{1}{j\omega C_{cx}} \tag{7}$$

- The parameters Z_1, Z_2 and Z_3 are obtained from the *Y*– parameters:

$$\boxed{Z_1 = \frac{1}{Y_{11} + Y_{12}}\ ; \qquad Z_4 = -\frac{1}{Y_{12}}\ ,}$$

$$\boxed{Z_3 = \frac{Y_{21} + Y_{11}}{(Y_{11} + Y_{12}).(Y_{22} + Y_{12})}\ .}$$

C. Determination of (Rbi/Rbc) and (Rbi.Cbc)



$$K_{Z13} = \frac{Z_1}{Z_3} = \frac{R_{bi}}{R_{bc}}\left(1 + j\omega R_{bc} C_{bc}\right)$$

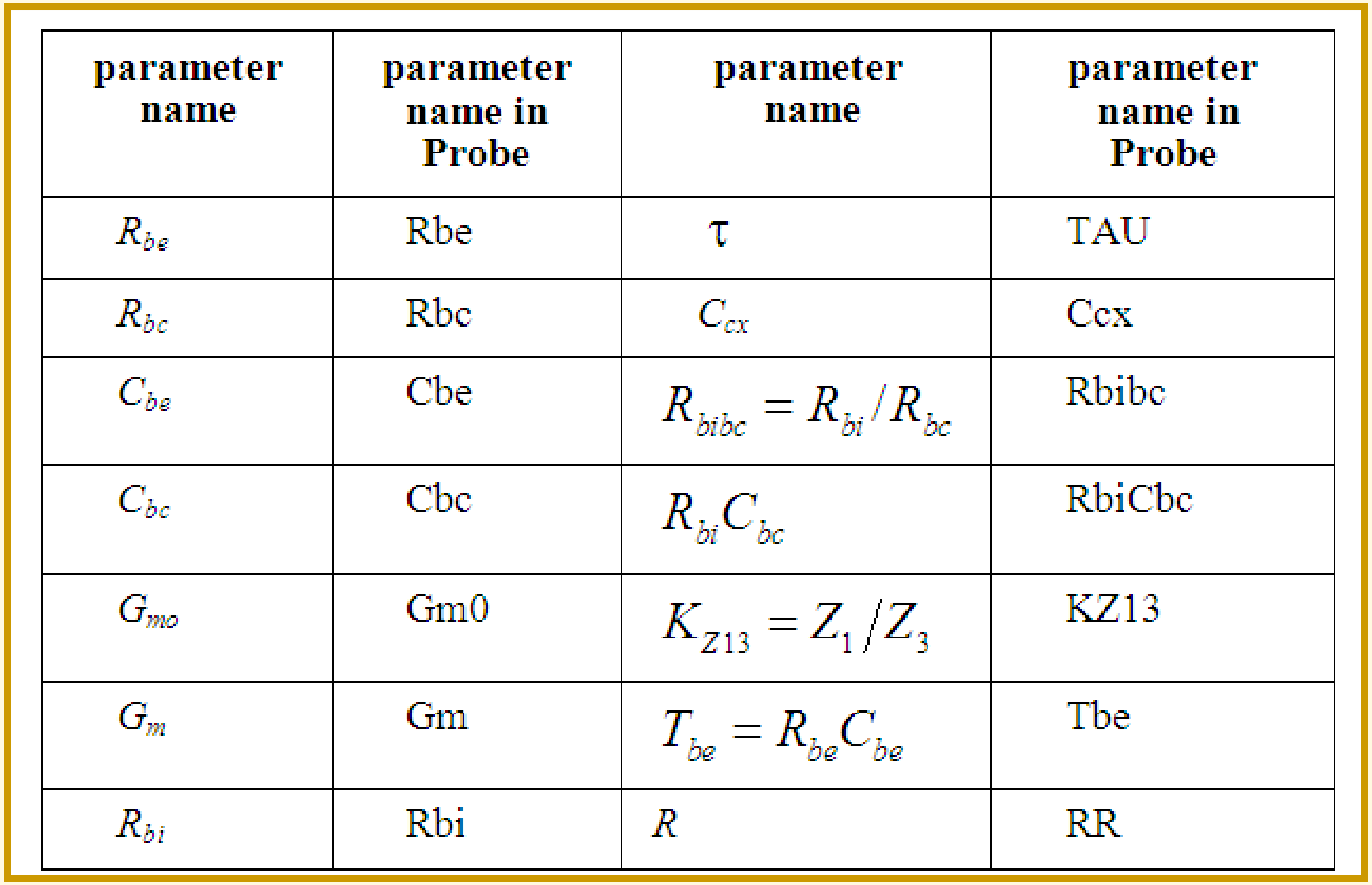
$$K_{Z13} = \frac{Z_1}{Z_3} = \frac{Y_{22} + Y_{12}}{Y_{21} + Y_{11}}$$

$$\frac{R_{bi}}{R_{bc}} = \mathrm{Re}\left(\frac{Z_1}{Z_3}\right)$$

$$R_{bi} C_{bc} = \frac{1}{\omega}\mathrm{Im}\left(\frac{Z_1}{Z_3}\right)$$

- The extraction procedure is realized using macro-definitions in the graphical analyzer *Cadence Probe*.
- The correspondence of the model parameter names and the names used in *Probe*, is given in Table I.

Table I



parameter name	parameter name in Probe	parameter name	parameter name in Probe
R_{be}	Rbe	τ	TAU
R_{bc}	Rbc	C_{cx}	Ccx
C_{be}	Cbe	$R_{bibc} = R_{bi} / R_{bc}$	Rbibc
C_{bc}	Cbc	$R_{bi} C_{bc}$	RbiCbc
G_{mo}	Gm0	$K_{Z13} = Z_1 / Z_3$	KZ13
G_m	Gm	$T_{be} = R_{be} C_{be}$	Tbe
R_{bi}	Rbi	R	RR

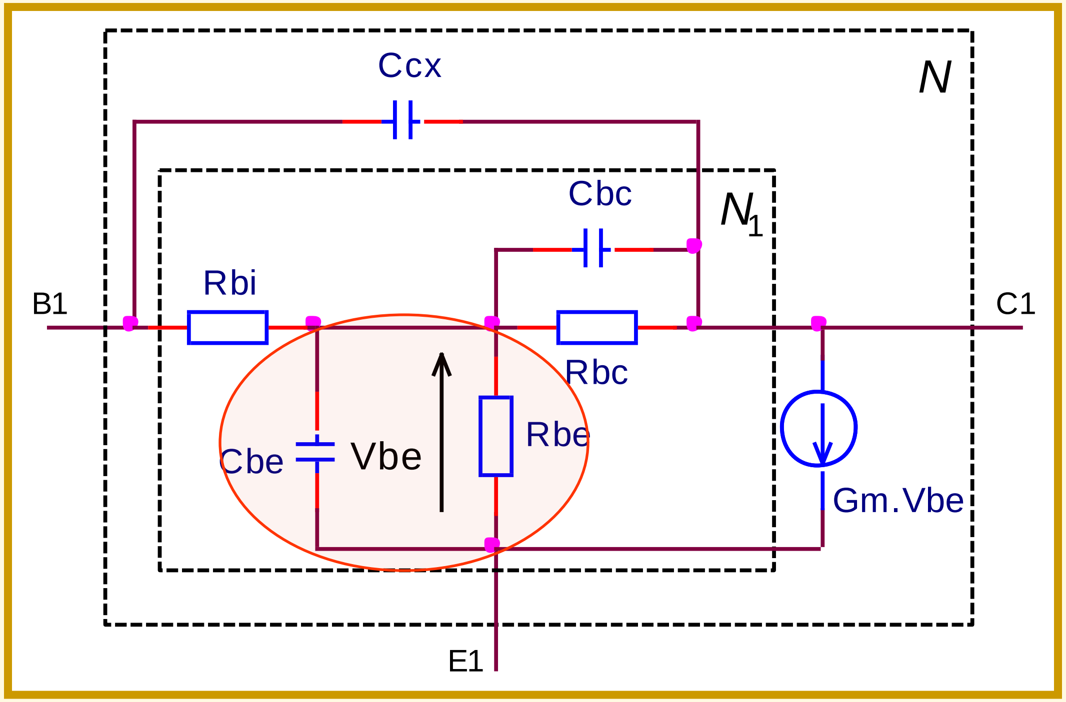
Determination of the ratio *(Rbi/Rbc)* using the *Probe* macros:

```
KZ13=(Y22+Y12)/(Y21+Y11)
Rbibc_f = R(Z13)
Rbibc = max(rms(Rbibc_f))
```

Determination of the product *(Rbi.Cbc)* using the macros:

```
w=2*pi*Frequency
RbiCbc_f = img(Z13)/w
RbiCbc =max(rms(RbiCbc_f))
```

D. Determination of (Rbe.Cbe)



Using (4) and (5), the following expression for Z_1 is obtained:

$$\boxed{Z_1 = \frac{R\left(1 + j\omega T\right)}{1 + j\omega T_{be}}} \qquad \boxed{R = R_{bi} R_{be}\left[\frac{1}{R_{bc}} + \frac{1}{R_{be}} + \frac{1}{R_{bi}}\right]} \qquad (13)$$

Im(Z_1) is obtained from (13):

$$\mathrm{Im}(Z_1) = \frac{R(T - T_{be})\omega}{1 + \omega^2 T_{be}^2};$$

$$F_1 = \frac{\omega}{\mathrm{Im}(Z_1)} = A + \omega^2 B;$$

$$A = \frac{1}{R(T - T_{be})}; \quad B = \frac{T_{be}^2}{R(T - T_{be})}.$$

A and *B* are determined using least squares fitting of F_1 as a function of the frequency.

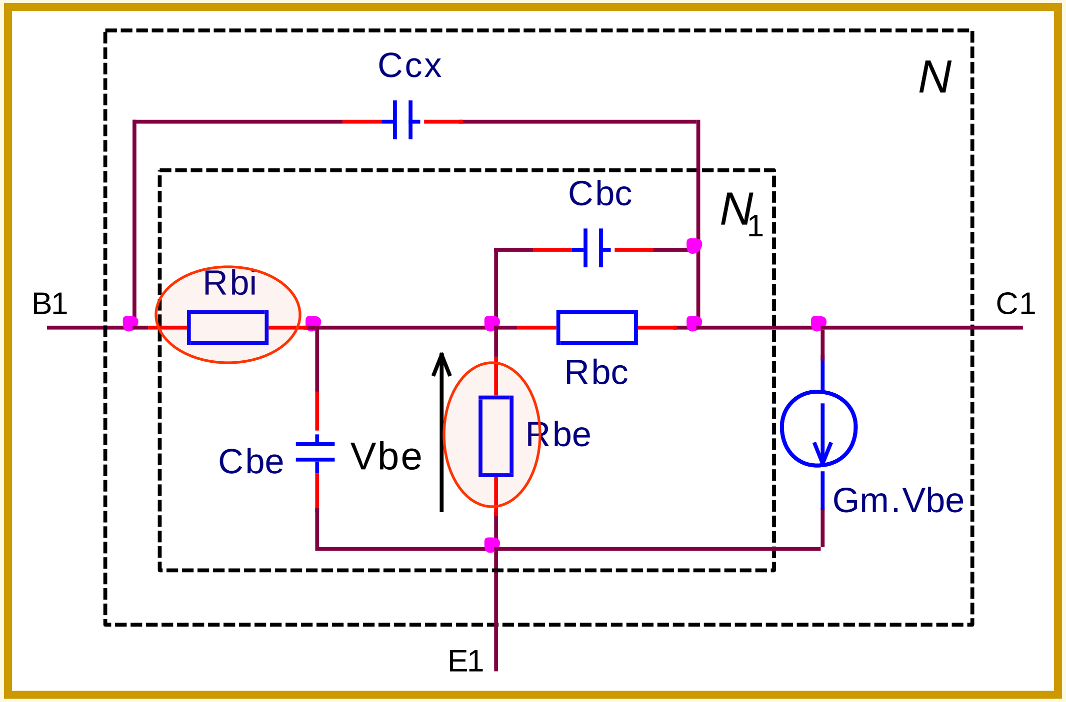
Tbe is obtained in the form:

$$R_{be} C_{be} = T_{be} = \sqrt{\frac{B}{A}}. \qquad (19)$$

Determination of the product (Rbe.Cbe) in *Probe*:

```
Z1=1/(Y11+Y12)
F1=w/IMG(Z1)
Fmax=max(frequincy)
wmax=2*pi*Fmax
B=MIN(F1)/wmax/wmax
A=min(F1-w*w*B)
Tbe=SQRT(B/A)
```

E. Determination of Rbe and Rbi



$$\boxed{F_2 = Z_1\left(1 + j\omega T_{be}\right) = R\left(1 + j\omega T\right)}$$

$$\boxed{R = \mathrm{Re}\left(F_2\right)}$$

$$\boxed{T = \frac{\mathrm{Im}\left(F_2\right)}{\omega R}}$$

R and *T* are determined applying least squares fitting of and as a function of the frequency using the following macros in *Probe* :

```
j=sqrt(-1)
F2=Z1*(1+j*w*Tbe)
RR_f=R(F2)
RR = max(rms(RR_f))
T_f=IMG(F2)/w/RR
T=max(rms(T_f))
```

Solving the linear system:

$$\begin{cases} R = CR_{be} + R_{bi} \\ RT = R_{bi}C_{bc}R_{be} + T_{be}R_{bi} \end{cases}$$

$$C = \left(1 + \frac{R_{bi}}{R_{bc}}\right)$$

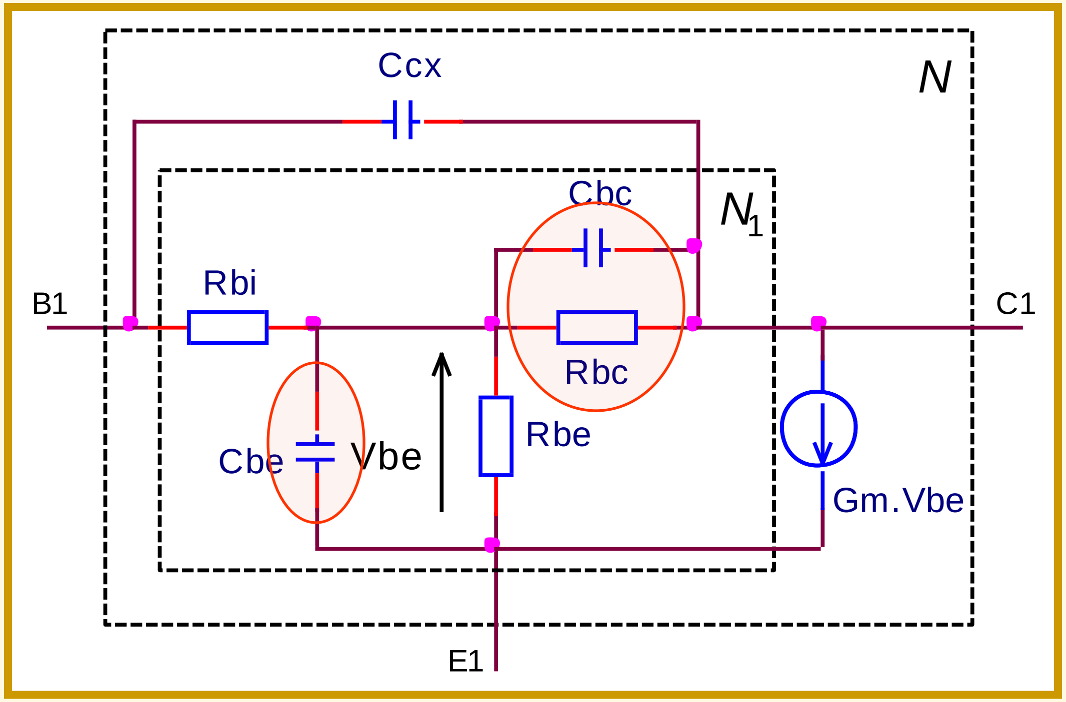
Rbe and Rbi are determined:

$$R_{be} = \frac{R(T - T_{be})}{R_{bi}C_{bc} - T_{be}C} \qquad R_{bi} = R - CR_{be} \qquad (23)$$

The following macros are defined for determination of Rbe and Rbi:

C=1+Rbibc
Rbe = RR*(T-Tbe)/(RbiCbc-C*Tbe)
Rbi = RR-C*Rbe

F. Determination of Rbc, Cbc and Cbe

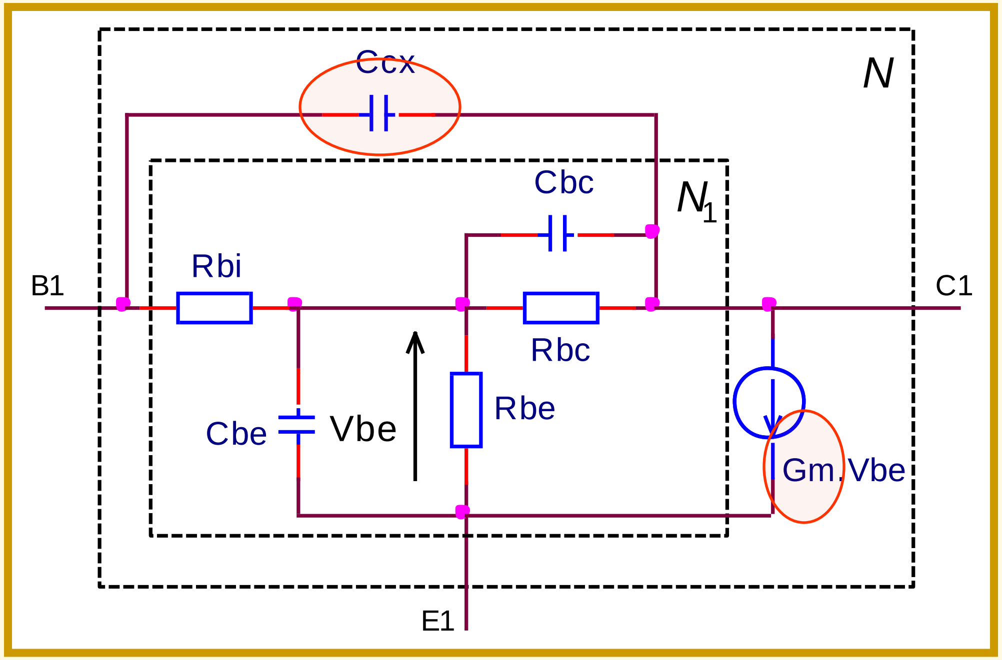


$$R_{bc} = \frac{R_{bi}}{R_{bibc}} \quad ; \quad C_{bc} = \frac{R_{bi}C_{bc}}{R_{bi}} \quad ; \quad C_{be} = \frac{T_{be}}{R_{be}} \; . \qquad (24)$$

The following macros are defined for determination of Rbc, Cbc and Cbe:

Rbc = Rbi/Rbibc
Cbc=RbiCbc/Rbi
Cbe=Tbe/Rbe

Determination of Ccx, Gmo and τ



$$Z_2 = \frac{Z_1 R_{bc}}{R_{be}}$$

$$Z_{cx} = \frac{Z_4 Z_2}{Z_2 - Z_4}$$

$$C_{cx} = \frac{1}{j\omega Z_{cx}}$$

Z2=Z1*Rbc/Rbe
Z3=Z1/KZ13
Z4=-1/Y12
Zcx=Z2*Z4/(Z2-Z4)
Ccx=min(img(1/(w*Zcx)))

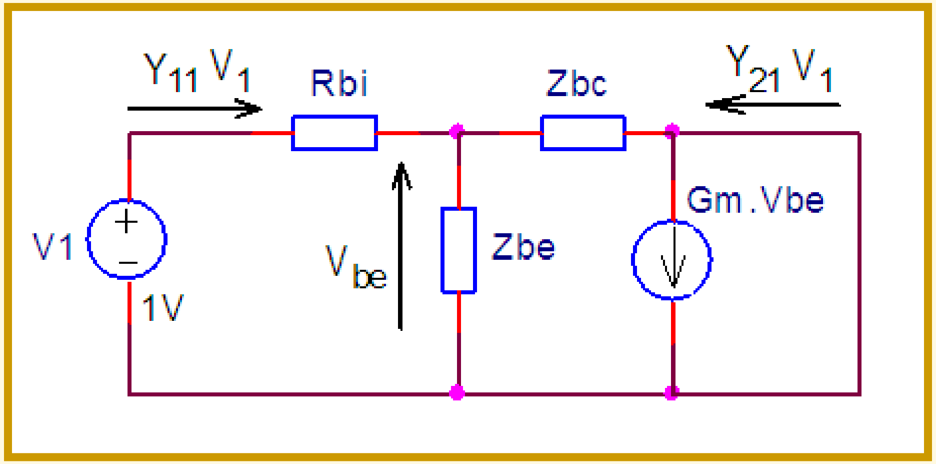


Fig. 5. Equivalent circuit for determination of *Gm*

$$V_{Rbi} = y_{11}.V_1.R_{bi}\,; \quad y_{21}.V_1 = G_m.V_{be} - \frac{V_{be}}{Z_{bc}}$$

$$V_{be} = V_1 - V_{Rbi} = V_1\left(1 - y_{11}.R_{bi}\right);$$

$$G_m = G_{mo}e^{-j\omega\tau}.$$

$$G_m = \frac{y_{21}}{1 - y_{11}.R_{bi}} + \frac{1}{Z_{bc}}$$

$$\tau = -\frac{phase(G_m)}{\omega}$$

Macros in *Probe*

```
Gm_f=Y21/(1-Y11*Rbi)-1/Zbc
Gm0=max(M(Y21/(1-Y11*Rbi)-1/Zbc))
TAU=max(TAU_f)
TAU_f=(Frequency/Fmax)*rms(-p(gm_f)/w/(180/pi))
```

τ

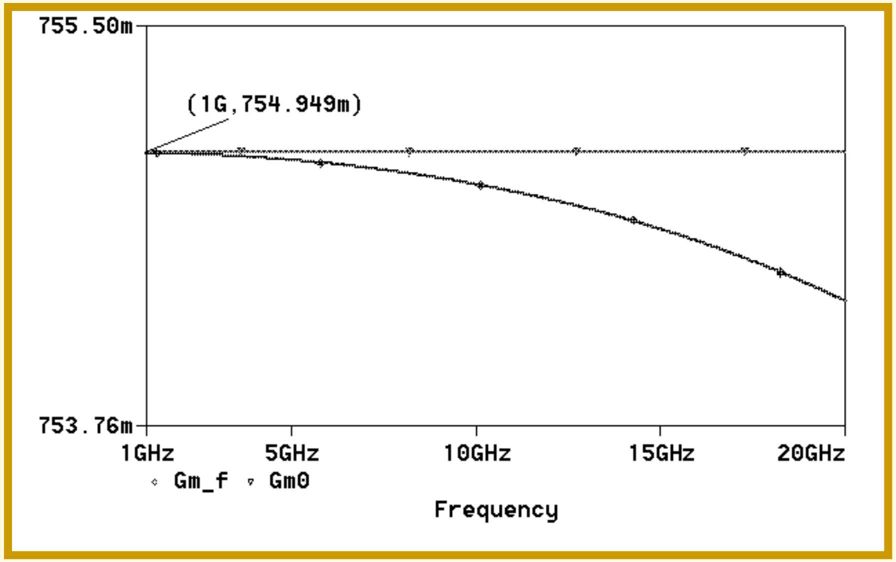


Fig. 6. Results for the extracted parameter Gm0

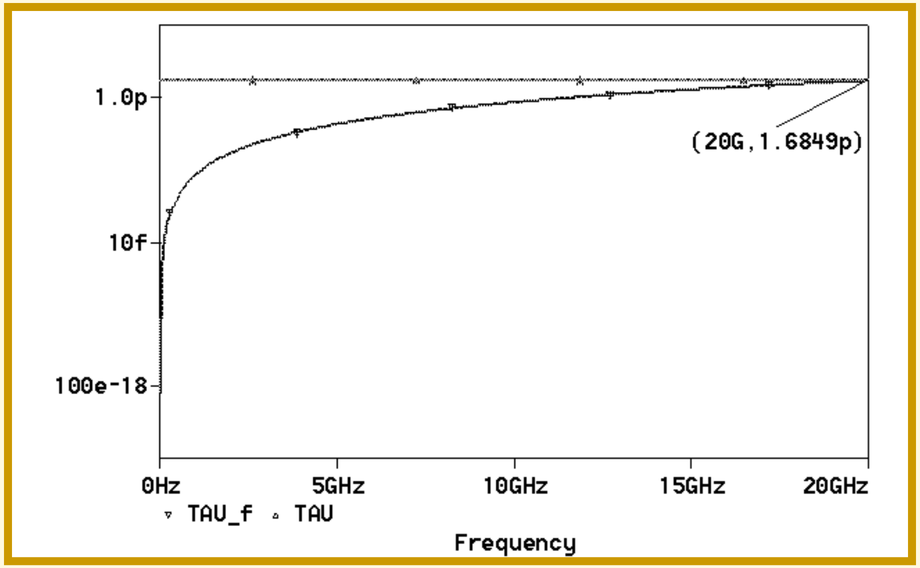


Fig. 7. Results for the extracted parameter τ

Example: Parameter extraction of small-signal HBT model based on measured data for *S*-parameters

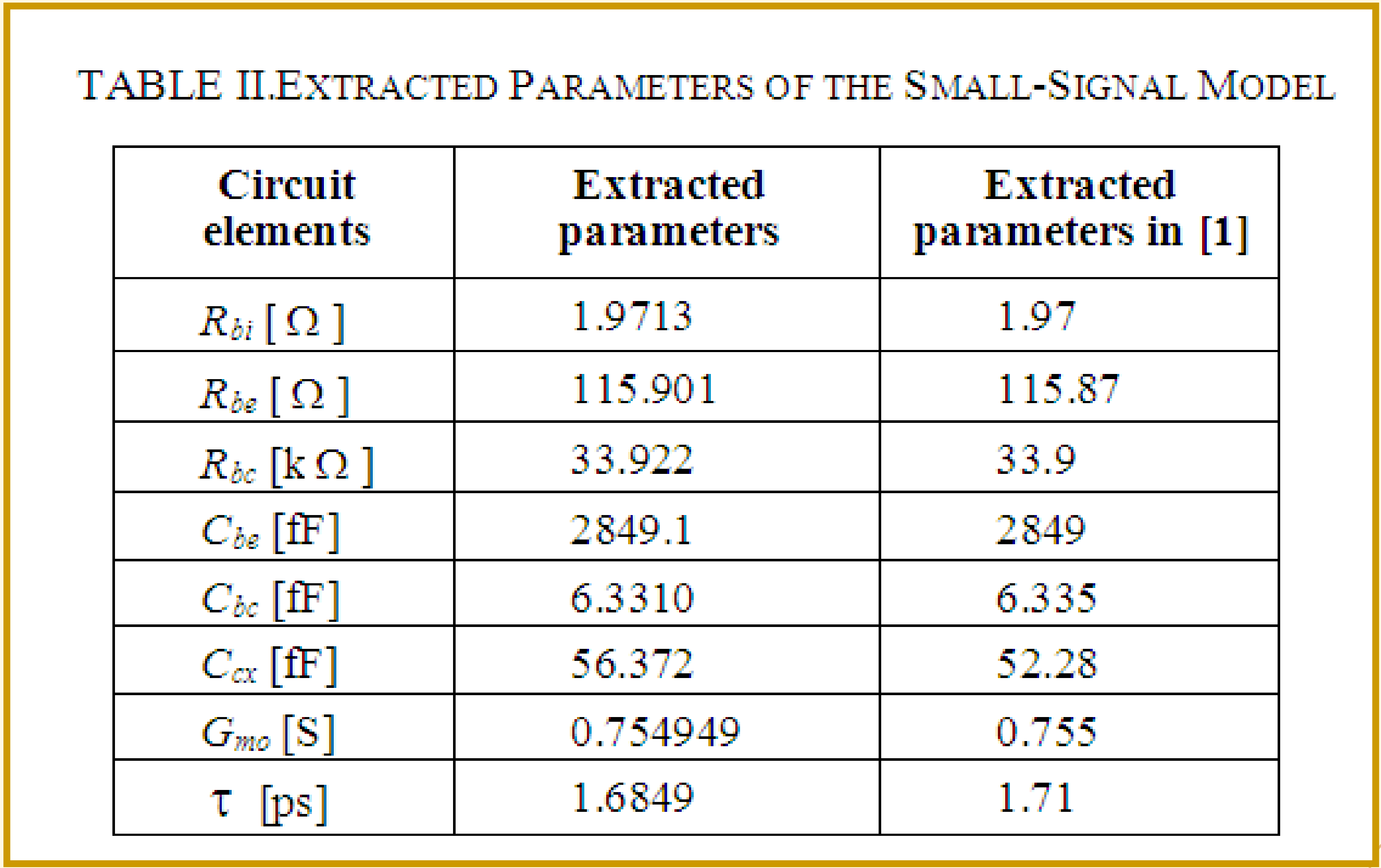
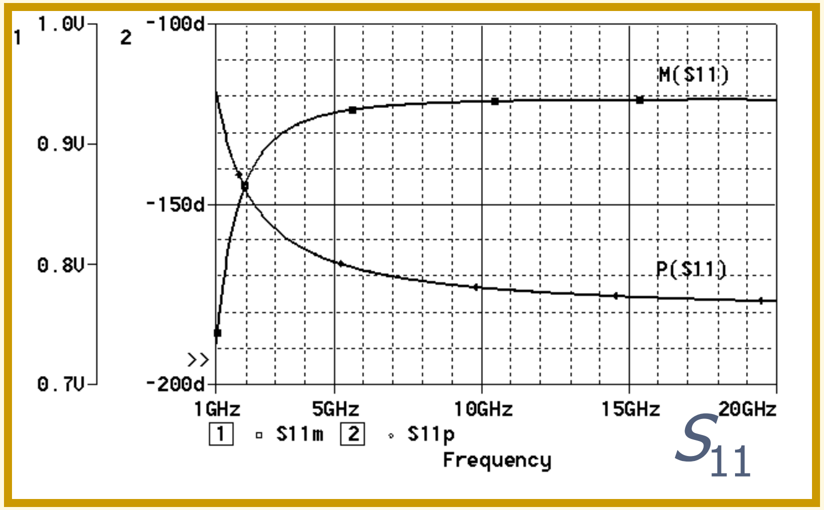
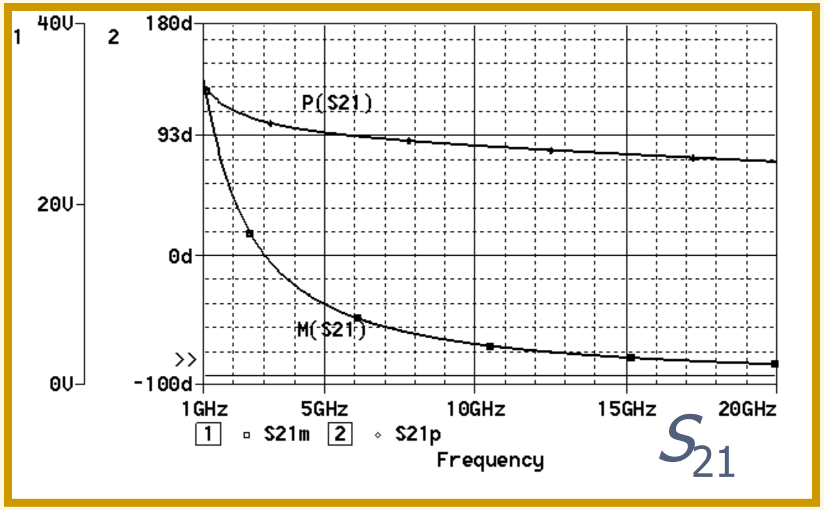


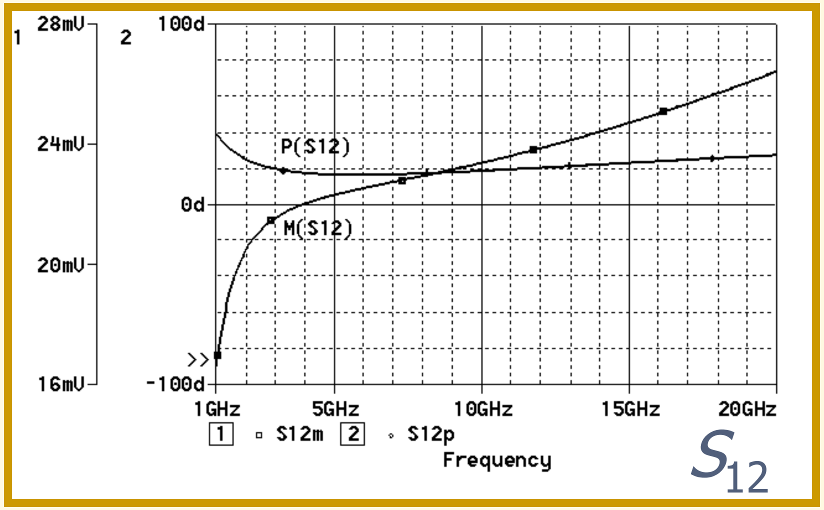
TABLE II. EXTRACTED PARAMETERS OF THE SMALL-SIGNAL MODEL

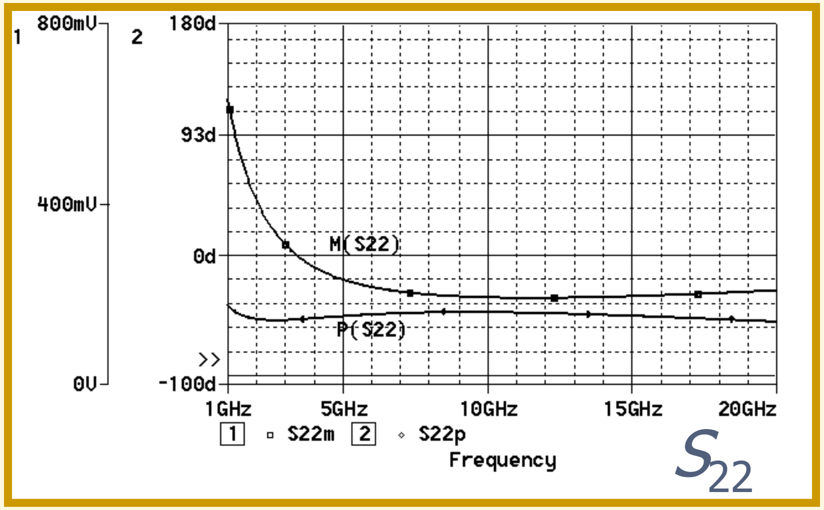
Circuit elements	**Extracted parameters**	**Extracted parameters in [1]**
R_{bi} [Ω]	1.9713	1.97
R_{be} [Ω]	115.901	115.87
R_{bc} [k Ω]	33.922	33.9
C_{be} [fF]	2849.1	2849
C_{bc} [fF]	6.3310	6.335
C_{cx} [fF]	56.372	52.28
G_{mo} [S]	0.754949	0.755
τ [ps]	1.6849	1.71

Simulation results for the S-parameters









The results are in a good agreement with the measured *S*-parameters presented in [1] over the frequency range 1-20GHz. **The relative error does not exceed 5%.**

CONCLUSIONS

- ❑ An approach to direct parameter extraction of small-signal HBT model has been developed, based on *S*-parameter measurements.
- ❑ The extraction procedure is realized in the Cadence Capture environment using the *Cadence PSpice* simulator and the graphical analyzer *Probe.* The computer realization is performed using macro-definitions in *Probe.*
- ❑ The obtained results are in a very good agreement with the measurement data.

Thank you for your attention!