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The difference between voltage and potential difference
Introduction (1)

- different definitions of potential difference, voltage and electromotive force → confusion with some basic notions

- there is a difference between voltage and potential difference, depending on what is our observation point

- static electric fields → conservative fields → the electromotive force for any closed curve is zero

- time-varying electric field is not a conservative field → the electromotive force induced in the closed curve can be expressed in terms of partial time derivative of the magnetic flux and it is different from zero
Introduction (2)

- what voltmeter measures, whether the position of observed points or position of the voltmeter leads affects the voltmeter readings?

- transmission line model → the voltage depends on the path of integration

- transversal voltage is a special case of voltage equal to the potential difference

- electrical circuit analysis → branch voltages are unique and equal to difference of nodal voltages (nodal potentials)
Static fields (1)

- static fields do not change with time → the simplest kind of fields
- electrostatic fields → produced by static electric charges
- stationary currents → associated with free charges moving along closed conductor circuits
- magnetostatic fields → due to motion of electric charges with uniform velocity (direct current) or static magnetic charges (magnetic poles)
- the electric field generated by a set of fixed charges can be written as the gradient of a scalar field → electric scalar potential $\varphi$

$$\vec{E} = -\nabla \varphi$$

$\vec{E}$ – electric field intensity
$\varphi$ – electric scalar potential
Static fields (2)

- unique voltage $u_{AB}$ can be defined for any pair of points $A$ and $B$ independent of the path of integration between them

$$u_{AB} = \int_{A}^{B} \vec{E} \cdot d\vec{l} = \varphi_A - \varphi_B \quad ; \quad \forall C_i$$

- Stokes theorem $\rightarrow$ Maxwell equation for static electric fields:

$$\oint_{C_i} \vec{E} \cdot d\vec{l} = \iint_{S_i} \left( \nabla \times \vec{E} \right) \cdot d\vec{S} = 0$$

$$\nabla \times \vec{E} = 0$$
Static fields (3)

- the work done on the particle when it is taken around a closed curve is zero, so the voltage around any contour $C_i$ can be written as:

$$u_{AA} = \oint_{C_i} \vec{E} \cdot d\ell = \varphi_A - \varphi_A = 0 \quad ; \quad \forall C_i$$
Time-varying fields (1)

- can be generated by accelerated charges or time-varying current

\[ \nabla \times \vec{E} = -\frac{d\vec{B}}{dt} = -\frac{\partial \vec{B}}{\partial t} + \nabla \times \left( \vec{v} \times \vec{B} \right) \rightarrow \text{Maxwell equation for time-varying fields} \]

\( \vec{v} \) – relative velocity between magnetic field and medium

\( \vec{B} \) – magnetic flux density

\[ \vec{B} = \nabla \times \vec{A} \quad \vec{A} \text{ - magnetic vector potential} \]

\[ \vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t} + \vec{v} \times \vec{B} \rightarrow \text{the electric field intensity for time-varying fields} \]

\[ \vec{E} = \vec{E}_{\text{stat}} + \vec{E}_{\text{ind}} \rightarrow \text{total electric field intensity} \]

\[ \vec{E}_{\text{stat}} = -\nabla \varphi \]

\[ \vec{E}_{\text{ind}} = \vec{E}_{\text{tr}} + \vec{E}_{m} = -\frac{\partial \vec{A}}{\partial t} + \vec{v} \times \vec{B} \]
Time-varying fields (2)

- Closed curves:

\[ u = \oint \vec{E} \cdot d\vec{l} = \oint \vec{E}_{\text{stat}} \cdot d\vec{l} + \oint \vec{E}_{\text{ind}} \cdot d\vec{l} \]

\[ = 0 \quad e = e_{\text{tr}} + e_m \]

\( e \) – induced electromotive force

- for any contour \( C_i \), voltage \( u \) is equal to induced electromotive force \( e \):

\[ u_{AA}^{C_i} = e_{AA}^{C_i} = \oint \vec{E} \cdot d\vec{l} = \oint \vec{E}_{\text{ind}} \cdot d\vec{l} \]

voltage and induced electromotive force depend on the integration path

- transformer electromotive force, \( e_{\text{tr}} \), can be expressed as negative of partial time derivative of the magnetic flux \( \Phi \) through the contour \( C_i \) over the surface \( S_i \):

\[ e_{\text{tr}} = -\frac{\partial}{\partial t} \int_{C_i} \vec{A} \cdot d\vec{l} = -\frac{\partial}{\partial t} \iint_{S_i} \vec{B} \cdot d\vec{S} = -\frac{\partial \Phi}{\partial t} \]
Time-varying fields (3)

- **Open curves**: voltage between any pair of points $A$ and $B$ can be defined as:

$$u_{AB} = \int_{A}^{B} \vec{E} \cdot d\vec{l} = \int_{A}^{B} \vec{E}_{\text{stat}} \cdot d\vec{l} + \int_{A}^{B} \vec{E}_{\text{ind}} \cdot d\vec{l}$$

$$u_{AB} = \varphi_A - \varphi_B + e_{AB}$$

- difference between time-varying voltage and potential difference is evident and these two concepts are not equivalent

- potential difference between any two points is independent of the integration path

- voltage and induced electromotive force between any two points are not equal and depend on the integration path
AC voltmeter reading (1)

- conventional circuit analysis without time-varying fields → Ohm law and Kirchhoff voltage law
- time-harmonic electromagnetic field → Ohm law and Kirchhoff voltage law extend with Faraday law
- the voltmeter readings are path dependent
- the measured voltage depends on the rate of change of magnetic flux through the surface defined by the voltmeter leads and the electrical network
- time-harmonic electrical network currents and current through the voltmeter, connected between points A and B, will induce a transformer electromotive force:

\[ \bar{\varepsilon} = - j \cdot \omega \cdot \bar{\Phi} \]

- \( \bar{\varepsilon} \) – phasor of the induced electromotive force
- \( \bar{\Phi} \) – phasor of the magnetic flux through the contour
AC voltmeter reading (2)

- Thevenin equivalent consists of Thevenin electromotive force and Thevenin impedance and represents the electrical network between points $A$ and $B$

- Thevenin electromotive force $E_T$, induced electromotive force $\varepsilon$, magnetic flux $\Phi$ and current through the voltmeter are phasors with magnitudes equal to effective values

- voltmeter reading is equal to effective value of voltage on voltmeter impedance

$$U_V = |\overline{U}_V| = |\overline{I} \cdot \overline{Z}_V| = \left| \frac{\overline{E}_T + \overline{\varepsilon}}{\overline{Z}_T + \overline{Z}_V + \overline{Z}_L} \right| \cdot \overline{Z}_V$$
Transmission line model (1)

- two-conductor transmission line model $\rightarrow$ voltage $u$ and current $i$ along the line:

$$-\frac{\partial u}{\partial x} = R \cdot i + L \cdot \frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial x} = G \cdot u + C \cdot \frac{\partial u}{\partial t}$$

- in time-varying electromagnetic field, voltage between two points depends on integrating path

$$u_{14} = \int_{1}^{4} \vec{E} \cdot d\vec{\ell} = \varphi_1 - \varphi_4 = u$$

$$u_{23} = \int_{2}^{3} \vec{E} \cdot d\vec{\ell} = \varphi_2 - \varphi_3 = u + \frac{\partial u}{\partial x} \cdot dx$$

- transversal voltage is a special case of voltage equal to the potential difference
Transmission line model (2)

- Single-conductor representation of the two-conductor transmission line of length $\ell$, with uniformly distributed per-unit-length parameters $R$, $L$, $C$ and $G$:

- Transversal voltages $u_1$ and $u_2$ are equal to the potentials $\varphi_1$ and $\varphi_2$
Electrical circuit theory (1)

- is an approximation of electromagnetic field theory that can be obtained from Maxwell equations

- active circuit elements: current and voltage sources

- passive circuit elements: resistance, inductance and capacitance

- in direct current, time-harmonic and transient electrical circuit analysis, voltage is unique and equal to difference of nodal voltages (nodal potentials)
Summary

- only in the **static fields**, voltage is identical to the potential difference (due to conservative nature of static fields, voltage does not depend on the integration path between any two points)

- in the **time-varying fields** → voltage and potential difference are not identical; potential difference between two points is unique; voltage and induced electromotive force depend on the integration path

- in the **transmission line model** → the time-varying voltage between two points depends on the path of integration → voltage is ambiguous; transversal voltage is a special case of voltage equal to the potential difference

- in **electrical circuit analysis** → voltage is unique and equal to difference of nodal voltages (nodal potentials)
Thank you!