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The difference between voltage and potential difference

Introduction (1)

- different definitions of potential difference, voltage and electromotive force → confusion with some basic notions

- there is a difference between voltage and potential difference, depending on what is our observation point

- static electric fields → conservative fields → the electromotive force for any closed curve is zero

- time-varying electric field is not a conservative field → the electromotive force induced in the closed curve can be expressed in terms of partial time derivative of the magnetic flux and it is different from zero

Introduction (2)

- what voltmeter measures, whether the position of observed points or position of the voltmeter leads affects the voltmeter readings?

- transmission line model → the voltage depends on the path of integration

- transversal voltage is a special case of voltage equal to the potential difference

- electrical circuit analysis → branch voltages are unique and equal to difference of nodal voltages (nodal potentials)

Static fields (1)

- static fields do not change with time → the simplest kind of fields

- electrostatic fields → produced by static electric charges

- stationary currents → associated with free charges moving along closed conductor circuits

- magnetostatic fields → due to motion of electric charges with uniform velocity (direct current) or static magnetic charges (magnetic poles)

- the electric field generated by a set of fixed charges can be written as the gradient of a scalar field → electric scalar potential φ

$$\vec{E} = -\nabla\varphi$$

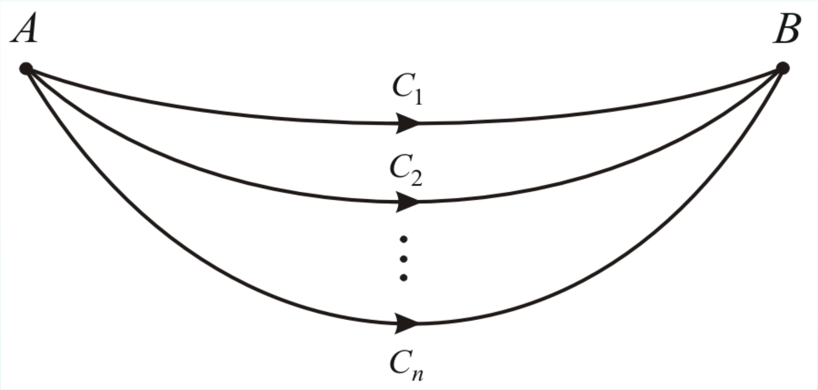
$\vec{E}$ – electric field intensity

φ – electric scalar potential

Static fields (2)

- unique voltage u_{AB} can be defined for any pair of points *A* and *B* independent of the path of integration between them

$$u_{AB} = \int_A^B \vec{E} \cdot d\vec{\ell} = \varphi_A - \varphi_B \quad ; \ \forall C_i$$



- Stokes theorem → Maxwell equation for static electric fields:

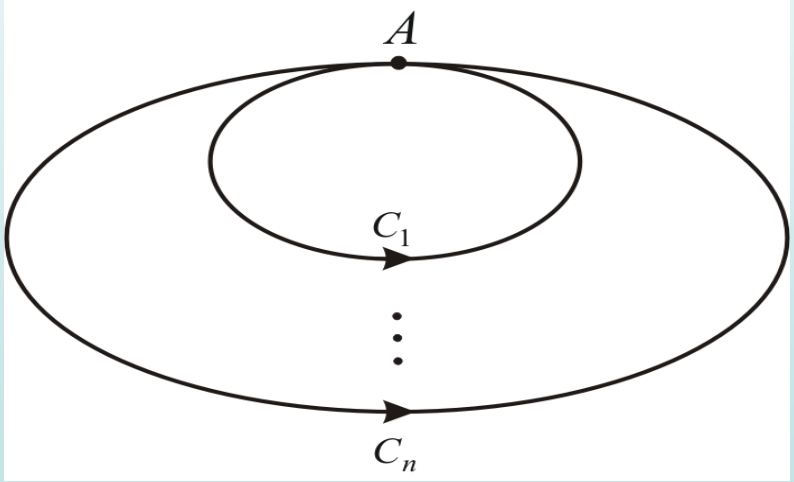
$$\oint_{C_i} \vec{E} \cdot d\vec{\ell} = \iint_{S_i} \left(\nabla \times \vec{E}\right) \cdot d\vec{S} = 0$$

$$\nabla \times \vec{E} = 0$$

Static fields (3)

➢ the work done on the particle when it is taken around a closed curve is zero, so the voltage around any contour C_i can be written as:

$$u_{AA} = \oint_{C_i} \vec{E} \cdot d\vec{\ell} = \varphi_A - \varphi_A = 0 \quad ; \quad \forall C_i$$



Time-varying fields (1)

- can be generated by accelerated charges or time-varying current

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt} = -\frac{\partial \vec{B}}{\partial t} + \nabla \times \left(\vec{v} \times \vec{B}\right)$$ → Maxwell equation for time-varying fields

$\vec{v}$ – relative velocity beetween magnetic field and medium

$\vec{B}$ – magnetic flux density

$$\vec{B} = \nabla \times \vec{A}$$

$\vec{A}$ - magnetic vector potential

$$\vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{\partial t} + \vec{v} \times \vec{B}$$ → the electric field intensity for time-varying fields

$$\vec{E} = \vec{E}_{stat} + \vec{E}_{ind}$$ → total electric field intensity

$$\vec{E}_{stat} = -\nabla\varphi$$

$$\vec{E}_{ind} = \vec{E}_{tr} + \vec{E}_{m} = -\frac{\partial \vec{A}}{\partial t} + \vec{v} \times \vec{B}$$

Time-varying fields (2)

- Closed curves: $$u = \oint_{C_i} \vec{E} \cdot d\vec{\ell} = \underbrace{\oint_{C_i} \vec{E}_{stat} \cdot d\vec{\ell}}_{=0} + \underbrace{\oint_{C_i} \vec{E}_{ind} \cdot d\vec{\ell}}_{=e=e_{tr}+e_m}$$

e – induced electromotive force

➢ for any contour C_i, voltage u is equal to induced electromotive force e:

$$u_{AA}^{C_i} = e_{AA}^{C_i} = \oint_{C_i} \vec{E} \cdot d\vec{\ell} = \oint_{C_i} \vec{E}_{ind} \cdot d\vec{\ell}$$

voltage and induced electromotive force depend on the integration path

➢ transformer electromotive force, e_{tr}, can be expressed as negative of partial time derivative of the magnetic flux Φ through the contour C_i over the surface S_i:

$$e_{tr} = -\frac{\partial}{\partial t} \oint_{C_i} \vec{A} \cdot d\vec{\ell} = -\frac{\partial}{\partial t} \iint_{S_i} \vec{B} \cdot d\vec{S} = -\frac{\partial \Phi}{\partial t}$$

Time-varying fields (3)

- Open curves: voltage between any pair of points A and B can be defined as:

$$u_{AB} = \int_A^B \vec{E} \cdot d\vec{\ell} = \underbrace{\int_A^B \vec{E}_{stat} \cdot d\vec{\ell}}_{=\varphi_A - \varphi_B} + \underbrace{\int_A^B \vec{E}_{ind} \cdot d\vec{\ell}}_{=e_{AB} = e_{trAB} + e_{mAB}}$$

$$u_{AB} = \varphi_A - \varphi_B + e_{AB}$$

➢ difference between time-varying voltage and potential difference is evident and these two concepts are not equivalent

➢ potential difference between any two points is independent of the integration path

➢ voltage and induced electromotive force between any two points are not equal and depend on the integration path

AC voltmeter reading (1)

- conventional circuit analysis without time-varying fields → Ohm law and Kirchhoff voltage law
- time-harmonic electromagnetic field → Ohm law and Kirchhoff voltage law extend with Faraday law
- the voltmeter readings are path dependent
- the measured voltage depends on the rate of change of magnetic flux through the surface defined by the voltmeter leads and the electrical network
- time-harmonic electrical network currents and current through the voltmeter, connected between points *A* and *B*, will induce a transformer electromotive force:

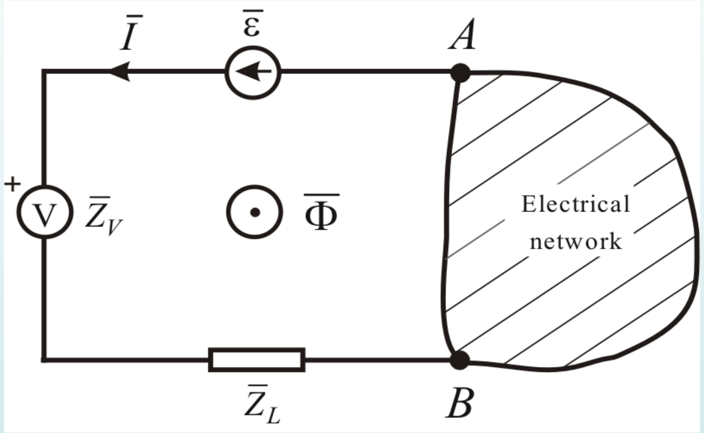
$$\bar{\varepsilon} = - j \cdot \omega \cdot \overline{\Phi}$$

$\bar{\varepsilon}$ – phasor of the induced electromotive force

$\overline{\Phi}$ – phasor of the magnetic flux through the contour

AC voltmeter reading (2)

➢ Thevenin equivalent consists of Thevenin electromotive force and Thevenin impedance and represents the electrical network between points *A* and *B*



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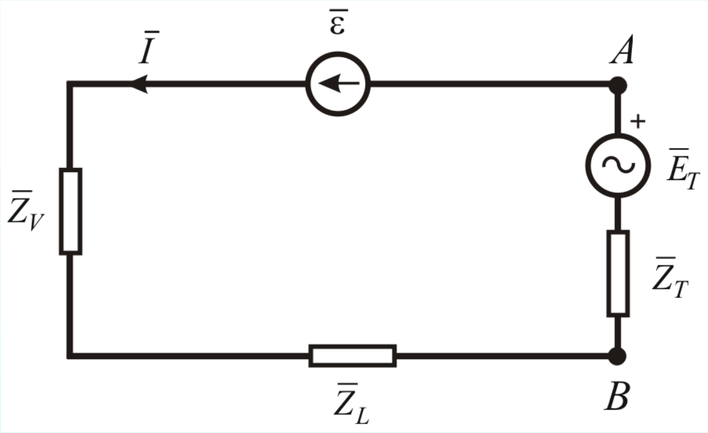


➢ Thevenin electromotive force E_T, induced electromotive force ε, magnetic flux Φ and current through the voltmeter are phasors with magnitudes equal to effective values

➢ voltmeter reading is equal to effective value of voltage on voltmeter impedance

$$U_V = \left|\bar{U}_V\right| = \left|\bar{I} \cdot \bar{Z}_V\right| = \left|\frac{\bar{E}_T + \bar{\varepsilon}}{\bar{Z}_T + \bar{Z}_V + \bar{Z}_L} \cdot \bar{Z}_V\right|$$

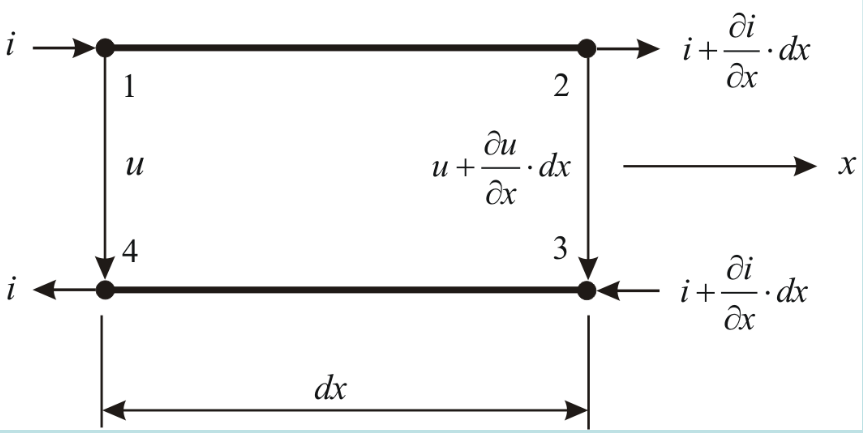
Transmission line model (1)

- two-conductor transmission line model → voltage *u* and current *i* along the line:

$$-\frac{\partial u}{\partial x} = R \cdot i + L \cdot \frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial x} = G \cdot u + C \cdot \frac{\partial u}{\partial t}$$

- in time-varying electromagnetic field, voltage between two points depends on integrating path



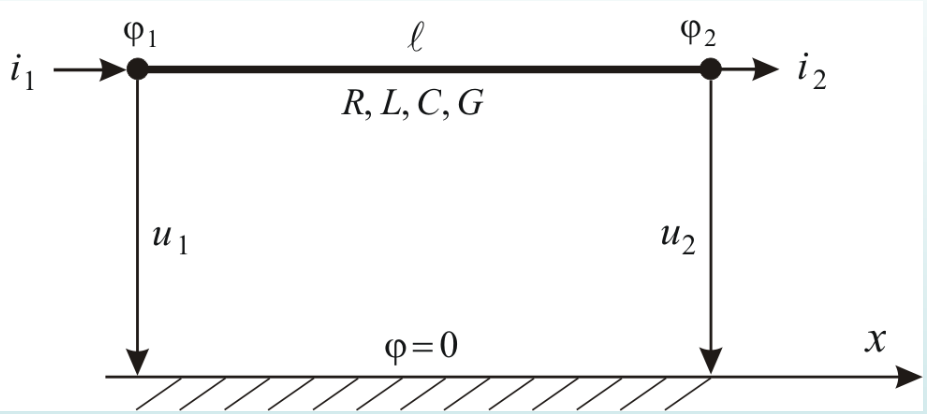
$$u_{14} = \int_1^4 \vec{E} \cdot d\vec{\ell} = \varphi_1 - \varphi_4 = u$$

$$u_{23} = \int_2^3 \vec{E} \cdot d\vec{\ell} = \varphi_2 - \varphi_3 = u + \frac{\partial u}{\partial x} \cdot dx$$

- <u>transversal voltage</u> is a special case of voltage equal to the potential difference

Transmission line model (2)

➢ single-conductor representation of the two-conductor transmission line of length ℓ, with uniformly distributed per-unit-length parameters *R*, *L*, *C* and *G*:



➢ transversal voltages u_1 and u_2 are equal to the potentials φ_1 and φ_2

Electrical circuit theory (1)

- is an approximation of electromagnetic field theory that can be obtained from Maxwell equations

- active circuit elements: current and voltage sources

- passive circuit elements: resistance, inductance and capacitance

- in direct current, time-harmonic and transient electrical circuit analysis, voltage is unique and equal to difference of nodal voltages (nodal potentials)

Summary

- only in the static fields, voltage is identical to the potential difference (due to conservative nature of static fields, voltage does not depend on the integration path between any two points)

- in the time-varying fields → voltage and potential difference are not identical; potential difference between two points is unique;
voltage and induced electromotive force depend on the integration path

- in the transmission line model → the time-varying voltage between two points depends on the path of integration → voltage is ambiguous
transversal voltage is a special case of voltage equal to the potential difference

- in electrical circuit analysis → voltage is unique and equal to difference of nodal voltages (nodal potentials)

Thank you!