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Ground Impedance of Cylindrical Metal Plate Buried in Homogeneous Earth

Introduction and brief outline of the paper

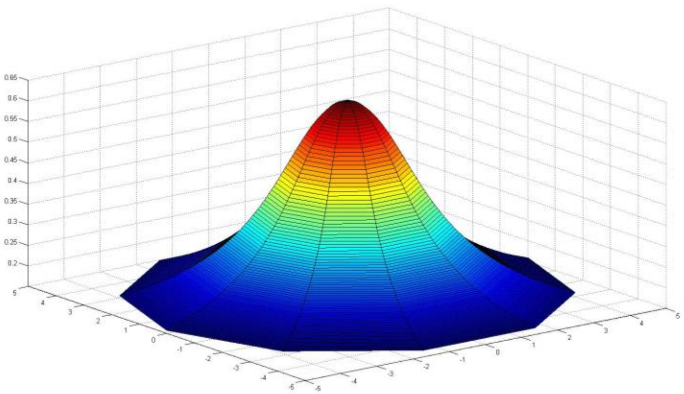
- cylindrical metal plates as grounding systems are usually used in ordinary households and offices
- plate ground impedance must be accurately computed (EMC applications, grounding analysis, ground fault current computation)
- ground impedance of plate buried in **homogeneous earth** will be derived using Galerkin-Bubnov weighted residual method
- combination of analytical and numerical integration will be used
- numerical examples

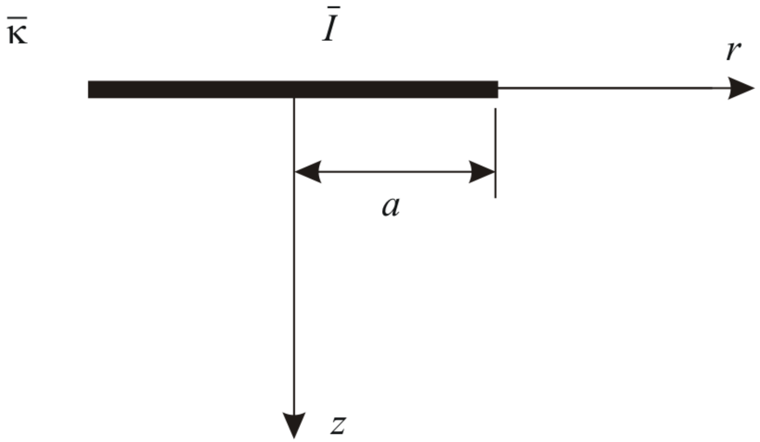
Metal plate in homogeneous and unbounded medium

- scalar electric potential distribution of **equipotential** metal plate obtained by extending the expression derived by Koch to include A.C. effects

$$\bar{\varphi} = \frac{\bar{I}}{j \cdot 8 \cdot \pi \cdot a \cdot \bar{\kappa}} \cdot \ell n \frac{\sqrt{r^2 + \left(|z| + j \cdot a\right)^2} + |z| + j \cdot a}{\sqrt{r^2 + \left(|z| - j \cdot a\right)^2} + |z| - j \cdot a}$$

$$\bar{\kappa} = \sigma + j \cdot 2 \cdot \pi \cdot f \cdot \varepsilon_0 \cdot \varepsilon_r$$





Metal plate in homogeneous and unbounded medium

➢ due to numerical stability reasons practical to transform the expression into:

$$\overline{\varphi} = \frac{\bar{I}}{4 \cdot \pi \cdot a \cdot \overline{\kappa}} \cdot \tan^{-1} \frac{a}{\alpha} \qquad \text{or} \qquad \overline{\varphi} = \frac{\bar{I}}{4 \cdot \pi \cdot a \cdot \overline{\kappa}} \cdot \tan^{-1} \frac{\beta}{|z|}$$

$$\alpha = \alpha(a, r, z) = \sqrt{\frac{A + \sqrt{A^2 + 4 \cdot a^2 \cdot z^2}}{2}}$$

$$A = r^2 + z^2 - a^2$$

$$\beta = \beta(a, r, z) = \sqrt{\frac{-A + \sqrt{A^2 + 4 \cdot a^2 \cdot z^2}}{2}}$$

Metal plate in homogeneous and unbounded medium

- metal plate potential can be obtained by introducing $r = 0$ and $z = 0$ into

$$\overline{\varphi} = \frac{\bar{I}}{4 \cdot \pi \cdot a \cdot \overline{\kappa}} \cdot \tan^{-1} \frac{a}{\alpha(a, r, z)}$$

- metal plate potential:

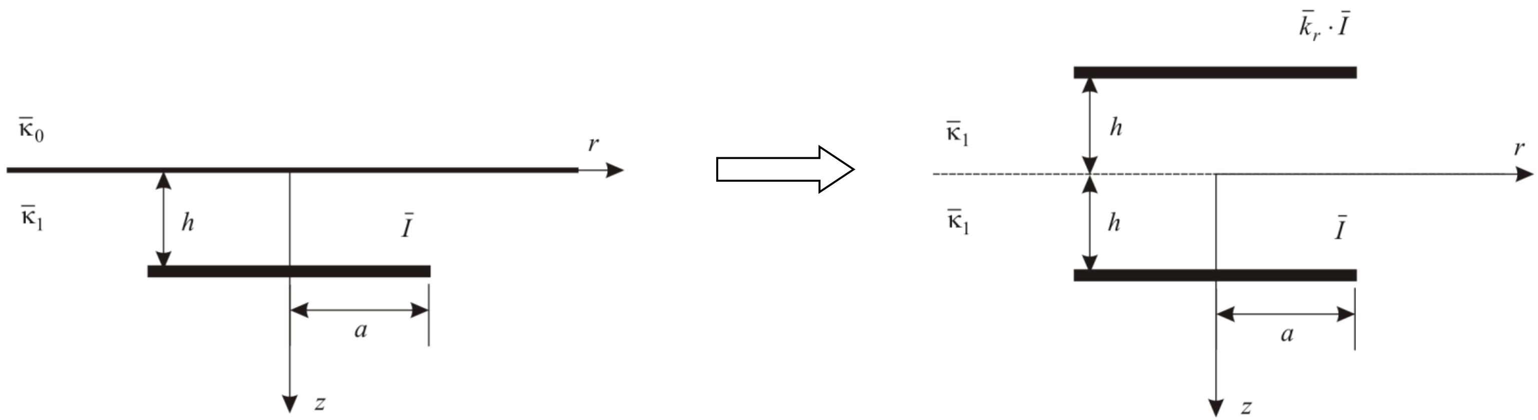
$$\overline{\Phi}_{pu} = \frac{\bar{I}}{8 \cdot a \cdot \overline{\kappa}}$$

- metal plate ground impedance in **homogeneous and unbounded** medium:

$$\bar{Z}_{pu} = \frac{1}{8 \cdot a \cdot \overline{\kappa}}$$

Metal plate in homogeneous earth

- two-layer medium is observed consisting of air and homogeneous earth



- scalar electric potential in homogeneous earth:

$$\bar{\varphi} = \frac{\bar{I}}{4 \cdot \pi \cdot a \cdot \bar{\kappa}_1} \cdot \tan^{-1} \frac{a}{\alpha(a, r, z-h)} + \frac{\bar{k}_r \cdot \bar{I}}{4 \cdot \pi \cdot a \cdot \bar{\kappa}_1} \cdot \tan^{-1} \frac{a}{\alpha(a, r, z+h)}$$

$$\bar{k}_r = \frac{\bar{\kappa}_1 - \bar{\kappa}_0}{\bar{\kappa}_1 + \bar{\kappa}_0}$$

Metal plate in homogeneous earth

- physically constant metal plate potential $\overline{\Phi}_p$ is approximated using Galerkin-Bubnov weighted residual method

$$\iint_{S_p} \left(\overline{\varphi}_p - \overline{\Phi}_p\right) \cdot N \cdot dS = 0$$

N – shape/weighting function

- shape function *N* is derived from the surface current density of a plate which leaks in homogeneous unbounded medium from both sides of the plate

$$\bar{J} = -2 \cdot \bar{\kappa} \cdot \lim_{\substack{|z| \to 0 \\ r \le a}} \frac{\partial \varphi}{\partial z} \quad \Longrightarrow \quad \bar{J} = \frac{\bar{I}}{2 \cdot \pi \cdot a \cdot \sqrt{a^2 - r^2}} = N \cdot \bar{I}$$

Metal plate in homogeneous earth

- approximation of the constant metal plate potential:

$$\overline{\Phi}_p = \iint_{S_p} \overline{\varphi}_p \cdot N \cdot dS = 0$$

- scalar potential for $z = h$ and $r \le a$:

$$\overline{\varphi}_p = \overline{P}(a, r, 0) \cdot \overline{I} + \overline{P}_m(a, r, 2 \cdot h) \cdot \overline{I}$$

$$\overline{P}(a, r, 0) = \frac{1}{4 \cdot \pi \cdot a \cdot \overline{\kappa}_1} \cdot \tan^{-1} \frac{a}{\alpha(a, r, 0)} = \frac{1}{8 \cdot a \cdot \overline{\kappa}_1}$$

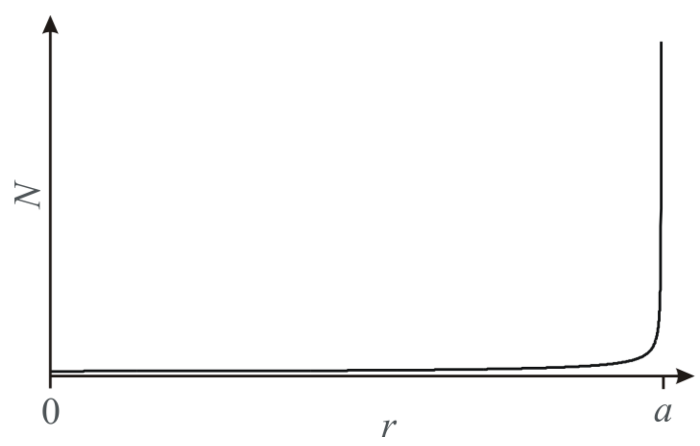
$$\overline{P}_m(a, r, 2 \cdot h) = \frac{\overline{k}_r}{4 \cdot \pi \cdot a \cdot \overline{\kappa}_1} \cdot \tan^{-1} \frac{a}{\alpha(a, r, 2 \cdot h)}$$

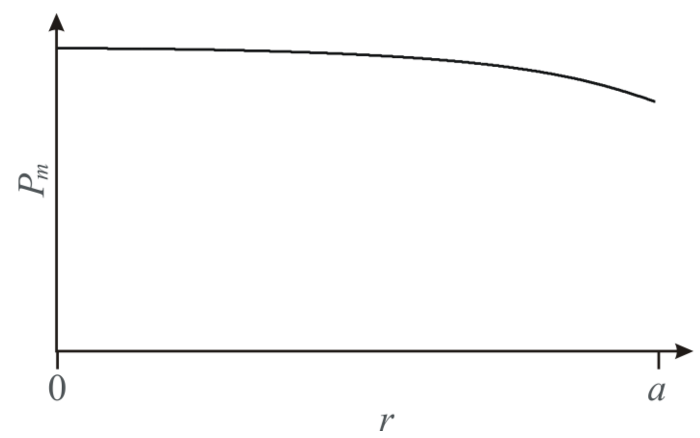
Metal plate in homogeneous earth

- approximation of the constant metal plate potential:

$$\overline{\Phi}_p = \bar{I} \cdot \left[\frac{1}{8 \cdot a \cdot \overline{\kappa}_1} + \int_0^a N \cdot \overline{P}_m(a, r, 2 \cdot h) \cdot 2 \cdot \pi \cdot r \cdot dr \right]$$

- combination of analytical and five-point Gaussian numerical integration





Metal plate in homogeneous earth

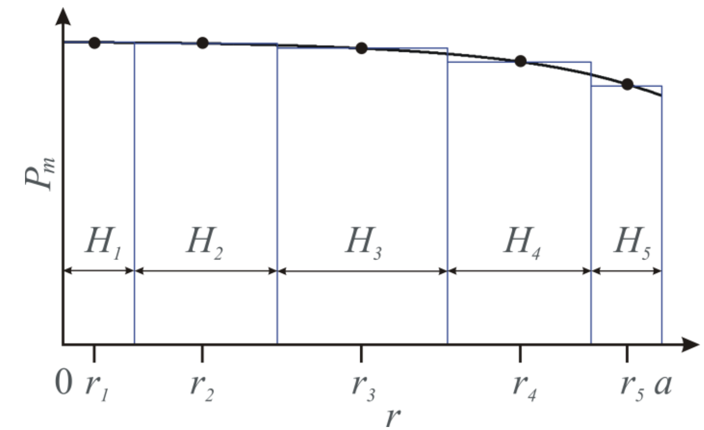
- combined integration yields

$$\overline{Z}_{mu} = \sum_{j=1}^{5} \overline{P}_m(a, r_j, 2 \cdot h) \cdot W_j$$

$$r_j = u_j \cdot a$$

$$W_j = \int_{r_{sj}}^{r_{ej}} N \cdot 2 \cdot \pi \cdot r \cdot dr$$

$$= \sqrt{1-\left(\frac{r_{sj}}{a}\right)^2} - \sqrt{1-\left(\frac{r_{ej}}{a}\right)^2}$$



j	Coordinates u_j	Weights H_j
1	$\frac{1}{2} - \frac{1}{6} \cdot \sqrt{5+2 \cdot \sqrt{\frac{10}{7}}}$	$\frac{322 - 13 \cdot \sqrt{70}}{1800}$
2	$\frac{1}{2} - \frac{1}{6} \cdot \sqrt{5-2 \cdot \sqrt{\frac{10}{7}}}$	$\frac{322 + 13 \cdot \sqrt{70}}{1800}$
3	$\frac{1}{2}$	$\frac{64}{225}$
4	$\frac{1}{2} + \frac{1}{6} \cdot \sqrt{5-2 \cdot \sqrt{\frac{10}{7}}}$	$\frac{322 + 13 \cdot \sqrt{70}}{1800}$
5	$\frac{1}{2} + \frac{1}{6} \cdot \sqrt{5+2 \cdot \sqrt{\frac{10}{7}}}$	$\frac{322 - 13 \cdot \sqrt{70}}{1800}$

Metal plate in homogeneous earth

- final expression for plate ground impedance in homogeneous earth:

$$\overline{Z}_p = \frac{1}{8 \cdot a \cdot \overline{\kappa}_1} + \sum_{j=1}^{5} \overline{P}_m(a, r_j, 2 \cdot h) \cdot W_j$$

- high accuracy and robustness achieved by analytically integrating the weighting function N which has a singularity at $r = a$

- easily extended to compute mutual impedances between different plates

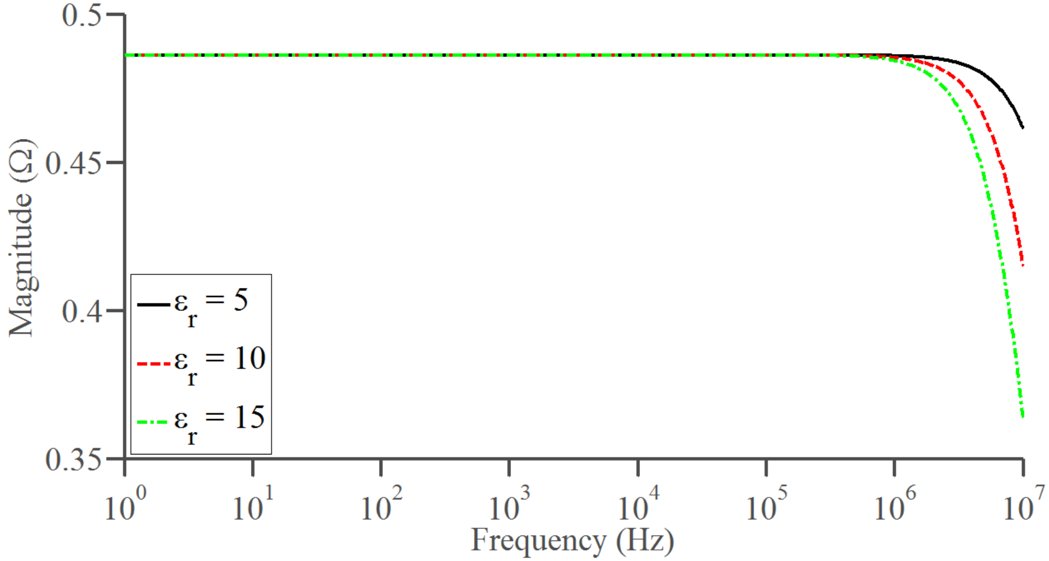
Numerical examples

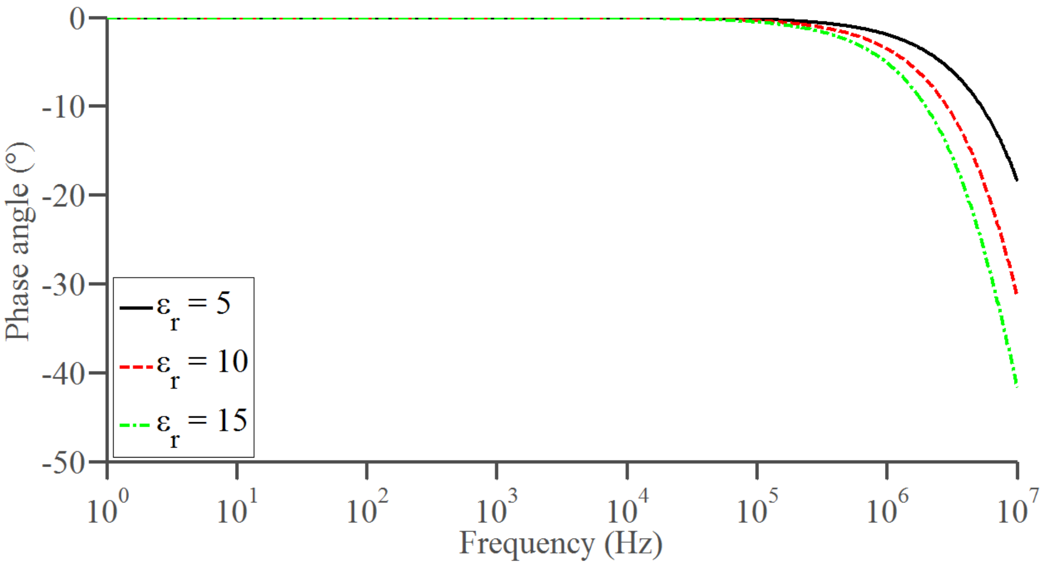
- metal plate of radius a = 50 m buried at depth h = 1 m is observed

- three types of earth with respect to conductivity observed:
  - high conductivity earth (0.01 S/m ↔ 100 Ωm)
  - medium conductivity earth (0.001 S/m ↔ 1 000 Ωm)
  - low conductivity earth (0.0001 S/m ↔ 10 000 Ωm)

- three types of earth with respect to relative permittivity observed:
  - $\varepsilon_r = 5$
  - $\varepsilon_r = 10$
  - $\varepsilon_r = 15$

High conductivity earth

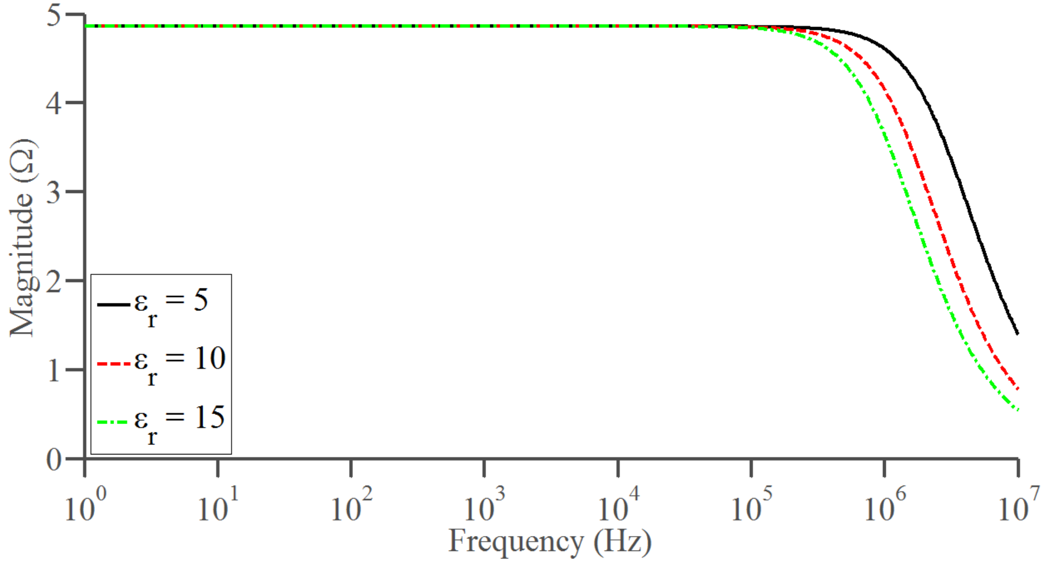




$$\bar{Z}_p = Z_p \cdot e^{j \cdot \varphi_p}$$

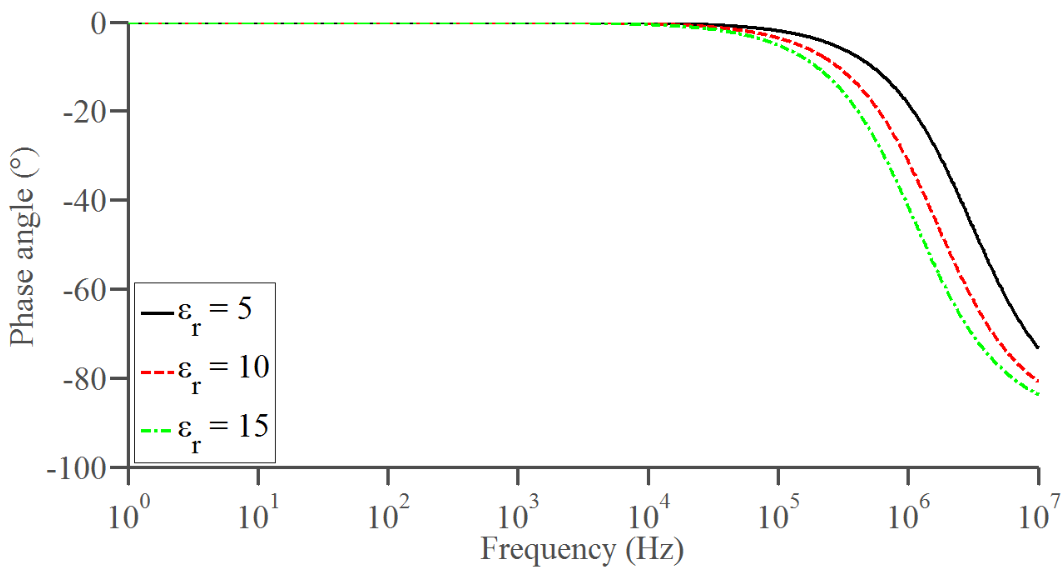
- magnitude starts to decay at high frequencies (1 MHz)
- phase angle tends to -90 ° at high frequencies
- ground impedance becomes capacitive at high frequencies
- decrease more pronounced for higher permittivity values

Medium conductivity earth



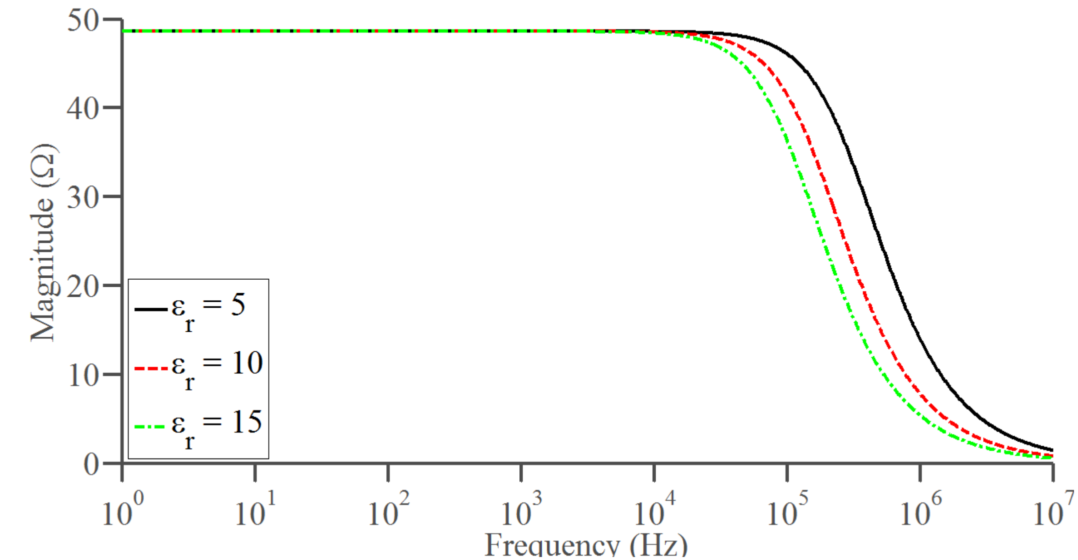
$$\overline{Z}_p = Z_p \cdot e^{j \cdot \varphi_p}$$

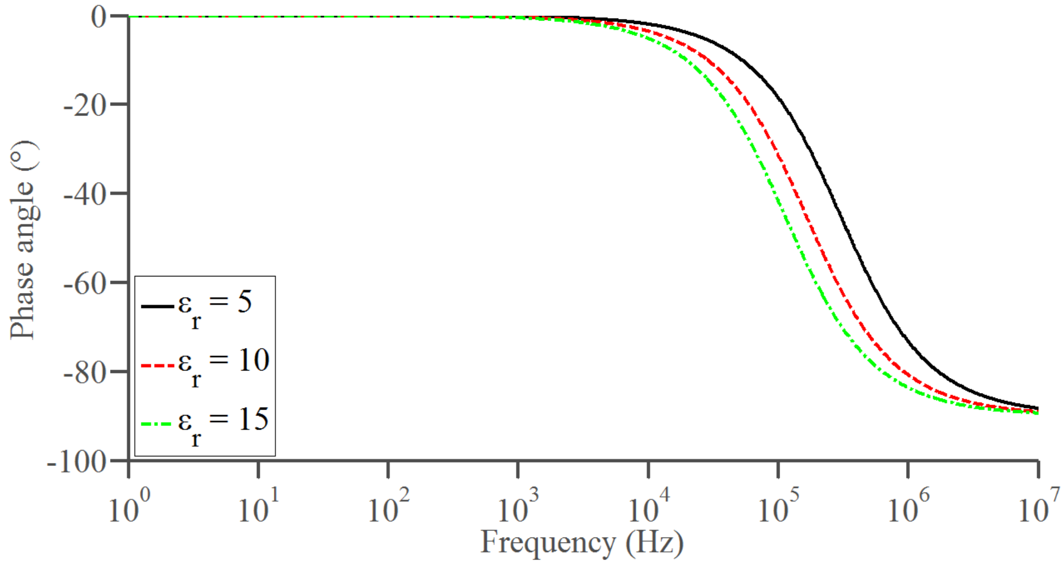
- magnitude starts to decay for $f \geq 100$ kHz



- ground impedance becomes capacitive at lower frequencies

Low conductivity earth





$$\overline{Z}_p = Z_p \cdot e^{j \cdot \varphi_p}$$

- magnitude starts to decay for $f \geq 10$ kHz
- capacitive character of ground impedance even lower frequencies

Summary

- highly accurate and robust algorithm for ground impedance of equipotential metal plate in homogeneous earth developed

- this is achieved by analytically integrating the weighting function N which has a singularity at $r = a$

- algorithm can be easily extended to obtain mutual impedances between different plates

- future research: extension of algorithm to accommodate a horizontal multilayer medium by the use of more sophisticated imaging methods

Thank you!